Basic probability distributions

$f = f(x)$ denotes PDF (probability density function),
in discrete distributions aka probability mass function: $f(x) = \Pr\{ X = x \}$.
$F = F(x)$ denotes CDF (cummulative density function).
E stands for the expected value (aka a mean).
$M = M(t) = \mathrm{E}(\exp(t\cdot X))$ – MGF (moment generating function) of the random variable X.

Discrete distributions: $F(x) = \Sigma_{X \le x} f(x)$, $\mathrm{E}(X) = \Sigma_x\, x\cdot f(x)$.

DiscreteUniform(n): $f(k) = 1/n$ for $k = 1, 2, \ldots, n$.
DiscreteTriangular(n).
Bernoulli(p): $f(k) = p$ for $k = 1$ (success), $= q := 1 - p$ otherwise, i.e., for $k = 0$ (failure); $0 < p < 1$.
Binomial(n, p): $f(k) = \Pr\{ X = k \} = \binom{n}{k} \cdot p^k \cdot q^{n-k}$, $k = 0, 1, 2, \ldots, n$: $M(t) = \{q+p\cdot\exp(t)\}^n$.

If p differs enough from 0 or to 1, and if the products $n\cdot p$ and $n\cdot q$ are big enough, then

$$f(x) \approx \frac{1}{\sqrt{2\pi}\cdot\sigma}\exp\left(-\frac{(k-\mu)^2}{2\sigma^2}\right)$$ with $\mu := n\cdot p$, $\sigma := n\cdot p\cdot q$.

(*Local Moivre-Laplace theorem*:) If $n\cdot p \to \mu$ and $n\cdot p\cdot(1-p) \to \sigma$ as $n \to \infty$,
then $\lim_{n\to\infty}$ Binomial(n, p) = Normal(μ, σ).

Geometric(p): $f(k) = q^{k-1}\cdot p$, $k = 1, 2, 3, \ldots$; $M(t) = p\cdot\exp(t)/\{1-q\cdot\exp(t)\}$ for $t < -\ln(1-p)$.
Poisson(λ): $f(k) = \lambda^k/k!\cdot\exp(-\lambda)$, $k = 0, 1, 2, \ldots$; $M(t) = \exp(\lambda\cdot\exp(t)-1)$.

Continuous distributions: $F(x) = \int_{t \le x} f(t)\, dt$, $\mathrm{E}(X) = \int_{-\infty}^{+\infty} t\cdot f(t)\, dt$.

Uniform(a, b): $f(x) = \frac{1}{b-a}$ if $x \in \langle a, b\rangle$, = 0 otherwise; $M(t) = \frac{\exp(b\cdot t)-\exp(a\cdot t)}{(b-a)\cdot t})$.

symmetric continuous **Triangular**(b), or shortly **Triangular**(b):
$f(x) = 1/b - |x|/b^2$ if $x \in \langle -b, b\rangle$, = 0 outside of $(-b, b)$; $b > 0$; $M(t) = \{\sinh(z)/z\}^2$ with $z := b/2\cdot t$.

Gauss(μ, σ), or **Normal**(μ, σ): $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\cdot\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$; $M(t) = \exp\left(\mu\cdot t + \frac{1}{2}\sigma^2 t^2\right)$

CDF of Gauss(0, 1) is $\Phi(x) = \frac{1}{\sqrt{2\pi}}\cdot\int_{-x}^{x}\exp\left(-\frac{t^2}{2}\right)dt = \frac{1}{2}\cdot\left\{1+\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right\}$,

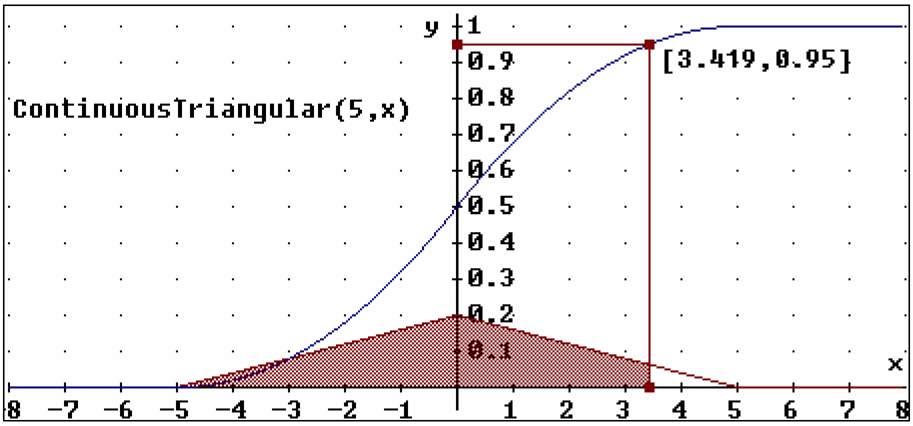
where the error function $\mathrm{erf}(a) = \frac{2}{\sqrt{\pi}}\cdot\int_0^a e^{-t^2}\, dt$.

CLT (*Central limit theorem*, aka *Lindenberg-Levy theorem*) in its classical version roughly says that if $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables, and they have the same expected value $\mathrm{E}(X_k) = \mu$ and the same finite variance $\mathrm{Var}(X_k) = \sigma^2$,
then $\lim_{n\to\infty} \Pr\left\{\sqrt{n}\cdot\left(\sum_{k=1}^{n} X_k - \mu\right) \le z\right\} = \Phi\left(\frac{z}{\sigma}\right)$.

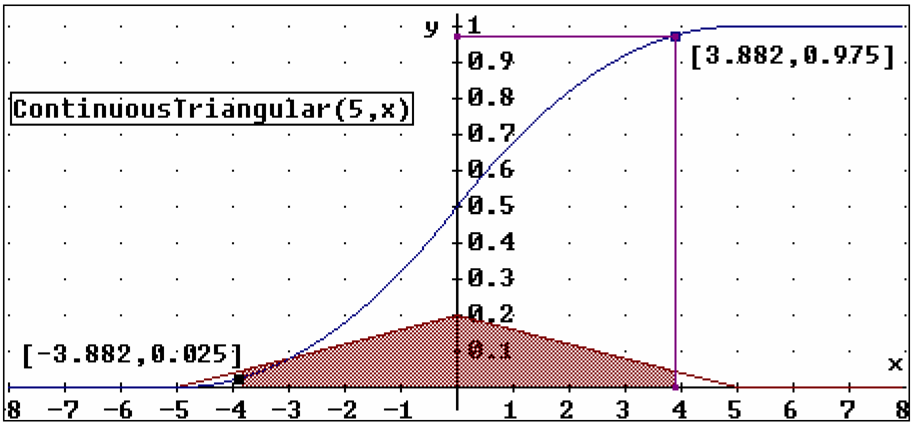
Gamma(k, s): $f(x) = \frac{1}{\Gamma(k)\cdot s^k}\cdot x^{k-1}\cdot e^{-\frac{x}{s}}$ for $x \ge 0$, = 0 for negative x; $M(t) = (1-s\cdot t)^{-k}$ for $t < 1/s$.

Gamma(1, $1/\lambda$) = **Exponential**(λ): $f(x) = \lambda\cdot\exp(-\lambda\cdot x)$ for $x \ge 0$, = 0 for $x < 0$;
$F(x) = 1 - \exp(-\lambda\cdot x)$; $\mathrm{E}(X) = \lambda$; $\mathrm{Var}(X) = \lambda^2$; $M(t) = 1/(1-t/\lambda)$.
Gamma(n, $1/\lambda$) = **Erlang**(k, λ), λ is called a rate parameter.
Gamma($m/2$, 2) = **ChiSquared**(m), m is called a DOF (degree of freedom); $M(t) = (1-2t)^{-m/2}$.

Cauchy: $f(x) = \{\pi\cdot(1+x^2)\}^{-1}$, $F(x) = 1/2+\arctan(x)/\pi$. X has no expected value $\mathrm{E}(X)$.



Symmetric continuous triangular distribution with the parameter 5, or shortly Triangular(5), distribution.
There are drawn graphs of PDF, f, and CDF, F of this distribution.
On the curve $y = F(x)$ there is marked the point $(5-\sqrt{10}/2, 0.95)$.
There is shaded the region $\{ (x, y) : x \leq 5-\sqrt{10}/2; 0 < y < f(x) \}$,
its area is $0.95 = 0.975-0 = F(5-\sqrt{10}/2) - F(-\infty) = \Pr \{ X \leq 0.95 \}$.
This interprets the right-tailed case.



Symmetric continuous triangular distribution with the parameter 5, or shortly Triangular(5), distribution.
There are drawn graphs of PDF, f, and CDF, F of this distribution.
On the curve $y = F(x)$ there are marked points $(-(5-\sqrt{5}/2), 0.025)$ and $(5-\sqrt{5}/2, 0.975)$.
There is shaded the region $\{ (x, y) : -(5-\sqrt{5}/2) < x \leq 5-\sqrt{5}/2; 0 < y < f(x) \}$,
its area is $0.95 = 0.975-0.025 = F(5-\sqrt{5}/2) - F(-(5-\sqrt{5}/2)) = \Pr \{ 0.025 < X \leq 0.975 \}$.
This interprets the both-tailed case.

Table of Triangular(5) CDF, $F = F(x)$,

the fragment covering the values $x \geq 2.7$

(more precisely: values $x = 2.6 + 0.1 \cdot j + 0.01 \cdot (k-1)$, where j runs the set $\{1, 2, \ldots, 24\}$ and k runs the set $\{1, 2, \ldots, 9\}$, so j corresponds tho the jth row, and k corresponds to the kth column of Table).

For example, $F(3.42) = 0.95$,

$F(4.293) = 0.990$

– all values are approximate, the linear interpolation between neighboring points is made if necessary (as here: $F(4.29) = 0.9899$, $F(4.30) = 0.9902$, the value increase by 0.9902–0.9899=0.003 takes place when the argument increases by 4.30–4.29=0.01).

	+0.00	+0.01	+0.02	+0.03	+0.04	+0.05	+0.06	+0.07	+0.08	+0.09
2.7	0.8942	0.8951	0.8960	0.8969	0.8978	0.8987	0.8996	0.9005	0.9014	0.9023
2.8	0.9032	0.9040	0.9049	0.9058	0.9066	0.9075	0.9084	0.9092	0.9101	0.9109
2.9	0.9118	0.9126	0.9134	0.9143	0.9151	0.9159	0.9167	0.9175	0.9183	0.9191
3	0.92	0.9207	0.9215	0.9223	0.9231	0.9239	0.9247	0.9255	0.9262	0.9270
3.1	0.9278	0.9285	0.9293	0.9300	0.9308	0.9315	0.9322	0.9330	0.9337	0.9344
3.2	0.9352	0.9359	0.9366	0.9373	0.9380	0.9387	0.9394	0.9401	0.9408	0.9415
3.3	0.9422	0.9428	0.9435	0.9442	0.9448	0.9455	0.9462	0.9468	0.9475	0.9481
3.4	0.9488	0.9494	0.9500	0.9507	0.9513	0.9519	0.9525	0.9531	0.9537	0.9543
3.5	0.955	0.9555	0.9561	0.9567	0.9573	0.9579	0.9585	0.9591	0.9596	0.9602
3.6	0.9608	0.9613	0.9619	0.9624	0.9630	0.9635	0.9640	0.9646	0.9651	0.9656
3.7	0.9662	0.9667	0.9672	0.9677	0.9682	0.9687	0.9692	0.9697	0.9702	0.9707
3.8	0.9712	0.9716	0.9721	0.9726	0.9730	0.9735	0.9740	0.9744	0.9749	0.9753
3.9	0.9758	0.9762	0.9766	0.9771	0.9775	0.9779	0.9783	0.9787	0.9791	0.9795
4	0.98	0.9803	0.9807	0.9811	0.9815	0.9819	0.9823	0.9827	0.9830	0.9834
4.1	0.9838	0.9841	0.9845	0.9848	0.9852	0.9855	0.9858	0.9862	0.9865	0.9868
4.2	0.9872	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0.9893	0.9896	0.9899
4.3	0.9902	0.9904	0.9907	0.9910	0.9912	0.9915	0.9918	0.9920	0.9923	0.9925
4.4	0.9928	0.9930	0.9932	0.9935	0.9937	0.9939	0.9941	0.9943	0.9945	0.9947
4.5	0.995	0.9951	0.9953	0.9955	0.9957	0.9959	0.9961	0.9963	0.9964	0.9966
4.6	0.9968	0.9969	0.9971	0.9972	0.9974	0.9975	0.9976	0.9978	0.9979	0.9980
4.7	0.9982	0.9983	0.9984	0.9985	0.9986	0.9987	0.9988	0.9989	0.9990	0.9991
4.8	0.9992	0.9992	0.9993	0.9994	0.9994	0.9995	0.9996	0.9996	0.9997	0.9997
4.9	0.9998	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
5	1	1	1	1	1	1	1	1	1	1

Values of Triangular(b) CDF, $F = F(x)$, for $x = 1, 2, 3, \ldots, 10$.
For example,
for $b = 5$ there is $F(2) = 0.82$,
for $b = 10$ there is $F(6) = 0.92$
and $F(7) = 0.955$,
so $F(6.86) = 0.950$
(6.86 is obtained by the linear interpolation:
the increase $\Delta x = 7–6 = 1$ causes the increase $\Delta F = 0.955–0.92 = 0.035$,
so the increase 0.95–0.92 = 0.03 is about 0.03/0.035 = 0.86
and, in consequence, it augments 6 to produce 6+0.86 = 6.86);
all values are approximate.

b	*x*=1	*x*=2	*x*=2	*x*=4	*x*=5	*x*=6	*x*=7	*x*=8	*x*=9	*x*=10
2	0.875	1	1	1	1	1	1	1	1	1
3	0.7777	0.9444	1	1	1	1	1	1	1	1
4	0.7187	0.875	0.9687	1	1	1	1	1	1	1
5	0.68	0.82	0.92	0.98	1	1	1	1	1	1
6	0.6527	0.7777	0.875	0.9444	0.9861	1	1	1	1	1
7	0.6326	0.7448	0.8367	0.9081	0.9591	0.9897	1	1	1	1
8	0.6171	0.7187	0.8046	0.875	0.9296	0.9687	0.9921	1	1	1
9	0.6049	0.6975	0.7777	0.8456	0.9012	0.9444	0.9753	0.9938	1	1
10	0.595	0.68	0.755	0.82	0.875	0.92	0.955	0.98	0.995	1
11	0.5867	0.6652	0.7355	0.7975	0.8512	0.8966	0.9338	0.9628	0.9834	0.9958
12	0.5798	0.6527	0.7187	0.7777	0.8298	0.875	0.9131	0.9444	0.9687	0.9861
13	0.5739	0.6420	0.7041	0.7603	0.8106	0.8550	0.8934	0.9260	0.9526	0.9733
14	0.5688	0.6326	0.6913	0.7448	0.7933	0.8367	0.875	0.9081	0.9362	0.9591
15	0.5644	0.6244	0.68	0.7311	0.7777	0.82	0.8577	0.8911	0.92	0.9444
16	0.5605	0.6171	0.6699	0.7187	0.7636	0.8046	0.8417	0.875	0.9042	0.9296
17	0.5570	0.6107	0.6608	0.7076	0.7508	0.7906	0.8269	0.8598	0.8892	0.9152
18	0.5540	0.6049	0.6527	0.6975	0.7391	0.7777	0.8132	0.8456	0.875	0.9012
19	0.5512	0.5997	0.6454	0.6883	0.7285	0.7659	0.8005	0.8324	0.8614	0.8878
20	0.5487	0.595	0.6387	0.68	0.7187	0.755	0.7887	0.82	0.8487	0.875
21	0.5464	0.5907	0.6326	0.6723	0.7097	0.7448	0.7777	0.8083	0.8367	0.8628

2014-05-28 Triangular(b).dfw